THE ISLAMIC CONNECTION

CONTRIBUTIONS
MADE BY
THE ARABS
TO
MATHEMATICS

by

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FOREWORD

This paper describes selected principal contributions to mathematics and the Arabs. Selection of appropriate examples to illustrate these contributions does not begin to approach the vast contributions made by the Arab Muslims during the five centuries from 700 CE to 1100 CE, known as the Muslim Golden Age.

By showing the Arab contribution to mathematics, the author has performed a great service in helping the reader to both understand and appreciate the vast and important contribution made by the Arab to mathematics.

Prof. Dr. Boris Kit
Frankfurt, Germany

DEDICATED

to

Kayla

The Arab Contribution to Mathematics

PREFACE

Most people in the United States have a very limited knowledge of the Middle East. They have a distorted and negative view. Also, they know little of the debt our mathematics owes Arabian medieval mathematicians.

I have included a map to which I occasionally make reference so the reader may see the locations. I have also included photographs of people, places and architecture germane to my account.

The Arab Contribution to Mathematics

MAP

The Geological Peninsula

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INTRODUCTION

This book is concerned with the Arabs, their contributions in mathematics. These translations transmitted knowledge to medieval Europe and are no less essential than original works, for had the research of mathematicians such as Aristotle, Euclid, Pythagoras, and Ptolemy been lost to posterity, the world would have been as poor as if they had never been produced.

What is unknown about the Arabs is much greater than what is known. There is as much misinformation as a lack of information concerning the Middle East, especially among those of us born, reared and educated in the United States. Other peoples and countries, on a level approaching the Arabs in historical interest and importance, have received much greater consideration and study in modern times than have they.

From the cradle of the Semitic family, the Arabian peninsula, these people, who later migrated into the Fertile Crescent and became the Babylonians, Assyrians, Phoenicians and the Hebrews of history. The deserts of the peninsula is where the element of Islam, Judaism and consequently of Christianity began.

In the sixth century of the common era [CE], Arabia gave birth to a people who conquered the civilized world and to Islam which claims nearly one billion people representing all races. Every fifth person in our world today is a follower of Islam. Islam - the religion, philosophy and culture of the Arabians - permeates all aspects of their lives. It is a living force and way of life to its adherents. Islam, like Judaism and Christianity, is the product of a spiritual life, the Semitic life. Within a century after the birth of Islam this empire and culture extended from the Atlantic Ocean to China, an empire greater than that of the Roman Empire at its zenith. They coupled their own civilization of the Babylonians with the Egyptians and the Greeks and acted as a medium for transmitting to Europe those intellectual influences which resulted in the Renaissance. No people in the Middle Ages contributed to human progress so much as did the Arabic speaking people. For over five centuries (700 CE - 1100 CE) during the Middle Ages more works were produced in philosophy, medicine, history, religion, science and mathematics through the medium of Arabic, than through any other language. The reader should keep in mind that within this book "Arabian" refers to an inhabitant of the **geological peninsula** **[See map]** and "Arab" for any Arabic-speaking person.

PART I

The Geological Arabian Peninsula: The Cradle of the Semites

The ***geological*** peninsula of Arabia is the south-western peninsula of Asia. The area exceeds a million square miles and comprises Bahrain, Iraq, Jordan, Kuwait, Lebanon, Oman, Palestine, Qatar, Saudi Arabia, Syria, United Arab Emirates, and Yemen. **(See map).** It is considered by many historians to have developed urban life under a common traditional patriarch long before written records existed. It was the cradle of the Semitic people - the Semites. These people became the Babylonians, the Assyrians, the Phoenicians and the Hebrews. Ibrahim (Abraham) came from the Ur of the Chaldees. One can visit the archeological site in Iraq today, located at 30.56 degrees North and 46.08 degrees East. **[24].**

The two surviving representatives of the Semitic people are the Arabians and the Hebrews. The Arabians have, to a larger degree than the Hebrews, preserved the characteristic physical features and mental traits of the Semitic family. Their language, Arabic, has conserved more of the peculiarities of the Semitic tongue - including the inflection - than the Hebrew.

We will later see how this Semite origin became a primary factor in the development of mathematics by the Arabs along the lines of their pragmatic philosophy.

In the United States the word “Semite” has come to possess a Jewish connotation. But to be “anti-Semitic” is to be unfavorable towards Arabs and Hebrews -- Muslims and Jews.

The term “Semite” comes from “Shem”, the eldest son of Noah (Gen. 10:1). **[28].**

Two major Semitic migrations began around 3500 before the common era [BCE] from the area of what is today Northern Saudi Arabia. One planted itself on top of the earlier Hamitic population of Egypt and the amalgamation produced the Egyptians of history.

A parallel migration struck root in the Tigro-Euphrates Valley, already populated by the Sumerians. The admixture of the two gave us the Babylonians.

Not long after the time of the Nile and Mesopotamia civilizations the southern part of Arabia, primarily what is today the country of Oman, became a populated commercial center. Precious metals, such as gold and copper which were mined in Arabia, became a common supply to northern civilizations along with gum resin, frankincense and myrrh. Commodities were transported by camel from the trading center, by caravan routes from Arabia to Egypt and Babylonia.

The Empty Quarter

The Iron Age
(2000 BCE - 200 CE)

The first recorded empire in history was that of Akkad in the area known as Babylonia, about 2369 BCE located in what is today called Iraq. **(See map).** Their influence spread beyond Mesopotamia to Arabia. However, after two centuries, it was overthrown by invading Amorites, from the present area of Syria, and who later became known as Aramaeans. A tribe from Asia Minor called Hittites conquered the Amorites in 1600 BCE and absorbed Syria. The barbarians from Europe overthrew the Hittites in 1200 BCE and were able to establish the Kassite Dynasty that ruled Babylonia successfully for four hundred years.

The Assyrians, who came from northern Mesopotamia thereafter ruled Babylonia. They introduced and developed cultural ideas of their own and these were assimilated with those of the Babylonians and Sumerians. They remained and ruled powerfully, employing iron weapons to arm their armies for several centuries, until they were overthrown by a combined force of Medes, Babylonians and Persians in 612 BCE. Nebuchadnezzer was the greatest King of the Babylonian Empire. He conquered Egypt, destroyed Jerusalem in 586 BCE and rebuilt Babylon which became one of the most impressive capitals the world has ever seen. In 539 BCE, his Empire fell to Cyrus, the Persian conqueror, and Nebuchadnezzer went into exile.

The Persians were able to conquer a vast region extending from Asia Minor, Egypt and even as far as India. About two centuries later, Alexander the Great finally conquered the Persian Empire in 323 BCE . He, however, died in Babylonia in the same year. The Sassanids were able to revive the Persian Empire during 226 common era [CE]. They were conquered by Moslem Arab armies in 640 CE through their ***Jihad*** or Holy Wars only 70 years after the Prophet Mohammed was born. **[14].**

The Rise of Islam

While this paper is about the history of mathematics between the years of 700 and 1200 C.E., it is impossible to discuss the contributions made to Mathematics by the Arabs without the mention of Islam.

The name of this religion is Islam. The word Islam means "peaceful submission." The root of which is ***Silm,*** which means the kind of peace represented by the opposite of war and ***Salam*** which means peace. Salam is also a greeting with peace. One of the many names of Allah (God) is that He is the Peace. It means submission to the One God, and to live in peace with the Creator, within one's self, with other people and with the environment. Thus, Islam is a total system of living. A Muslim is supposed to live in peace and harmony with all these segments.

The followers of Islam are called Muslims. Muslims are not to be confused with Arabs. Muslims may be Arabs or any other nationality.

An Arab could be a Muslim, a Christian, a Jew or an adherent to some other religion. Any person who adopts the Arabic language is called an Arab. However, the language of the Quran (the Holy Book of Islam) is Arabic. Muslims all over the world try to learn Arabic so that they may be able to read the Quran and understand its meaning. They pray in the language of the Quran, namely Arabic. Supplications to God could be in any language.

While there are one billion Muslims in the world there are about 200 million Arabs. Among them, less than ten percent, probably closer to five percent, are not Muslims. Thus Arab Muslims constitute only about twenty percent of the Muslim population of the world.

Muslims believe that Islam is a total and a complete way of life. It encompasses all aspects of life. As such, the teachings of Islam do not separate religion from science. As a matter of fact, science and religion, as well as politics and the state, are under the obedience of Allah through the teachings of Islam.

The Arab Contribution to Mathematics

After the subjugation of the North African coast as far as the Atlantic, the way was open for the conquest of the neighboring south-western part of Europe. [46, 47].

In 711 CE, Tariq Bin Ziad took the momentous step of crossing into Spain on a raid that developed into a conquest of the Iberian Peninsula (al-Andalus). The Goths were defeated and Andalusia was converted to Islam.

This constituted the last and most sensational of the major campaigns of the Arabs and by 732 CE resulted in the addition to the Moslem world the largest European territory ever held by them.

By the year 732 CE, marking the first centennial of the death of the Prophet Mohammed his followers were the masters of an empire greater than that of Rome at its zenith. The name of the Prophet Mohammed, son of Arabia, joined with the names of almighty Allah, was being called five times a day from across an empire extending over south-western Europe, northern Africa and western and central Asia.

After Mohammed died at age 63, leadership was passed to Abu Bakr in 632 A.D., who was the first caliph (khalifa) which means "successor."

Umar Ibn Al Khattab the 2nd Caliph until 611 C.E. The first four caliphs were from the Qumayyad family (Qoureish tribe) and their rule was centered in Damascus.

The 3rd caliph, Outhman, was murdered and his Ali, cousin of the Prophet Mohammed, became the fourth caliph. Mouaweeia, governor of Syria, accused Ali of complicity in the murder of Outhman and gained the caliphate after Ali was assassinated.

These rivalries resulted in the major division in the Muslem faith: the Sunni, who believed that the caliphate was an elective office and the Shi'a who believed that the heirs of Prophet Mohammed, namely his daughter Fatima and her husband Ali, were entitled to the caliphate.

The Qumayyard dynasty was overthrown in 750 CE by the descendants of Abbas. The Abbasid dynasty ruled from Baghdad until 1258 with the invasion of the Mongols. Prosperity reached its greatest peak during the Caliphate of Horoun Al-Rashid (786-809).

It was during the rule of Al-Rashid that the Arab Empire made its most significant achievements in mathematics.

Under Abu Bakr's leadership, General Khalid Ibn Al-Walid led the Arab Army in defeating the Byzantine and Sassonid Armies.

The Byzantine Army was defeated in 636 A.D. and Iraq and Persia between 637 & 650. Jerusalem was conquered in 638 and Egypt in 641. Conquests continued for a century expanding the Arab Empire from China to France.

The second stage [732 CE - 998 CE] of the expansion of Islam spread eastwards to the Sind Valley, Farghana and other areas bordering West China and westward to equatorial Africa.

The third stage occurred between 998 CE and 1243 CE, when Islam spread through India and Asia Minor. Among the reasons for the rapid and peaceful spread of Islam was the simplicity of its doctrine. Islam calls for faith in only One God worthy of worship. It also repeatedly instructs man to use his powers of intelligence and observation.

Within a few years, great civilizations and universities were flourishing, for according to the Prophet "***seeking knowledge is an obligation for every Muslim man and woman***." The synthesis of Eastern and Western ideas and of new thought with old, brought about great advances in medicine, mathematics, physics, astronomy, geography, architecture, art, literature, and history. Many crucial systems such as algebra, the Arabic numerals, and also the concept of the zero (vital to the advancement of mathematics), were transmitted to medieval Europe from Islam. Sophisticated instruments which were to make possible the European voyages of discovery were developed, including the astrolabe, the quadrant and good navigational maps. The prophet said, "***Seek knowledge even into china***…" The Hui Shen mosque was built in the seventh century.

Arab door with Islamic calligraphy

As stated earlier, this paper is about mathematics and history related to the Arabs; the continuity of knowledge through their original contributions and translations and the Muslem belief that all knowledge is sacred and leads ultimately to the knowledge of God.

We see these beliefs expressed in Islamic calligraphy - geometric, decorative and invocatory. These beliefs are reflected in their architecture - the gentle curves of the white arches, suggestive of the traditional ***madrasah*** or Mosque School, where learning and a closer relationship to the Creator were inseparable pursuits. **[23].**

Mathematics Found Within The Quran.

There are two major facets of the Quran's mathematical system: (1) The mathematical literary composition, and (2) The mathematical structure involving the numbers of suras and verses.

Finally, because this book is specially concerned with the connection between Islam and mathematics, the following mathematics found within the Quran itself is included without commentary.

Consider the first verse of the Quran:

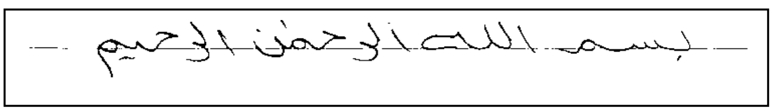


Figure 1

"In the Name of Allah, the Most Gracious, the Most Merciful."

The Arabic letters of this verse number nineteen (19):

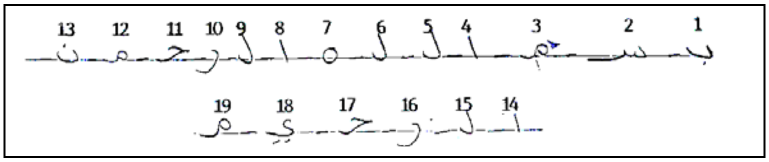


Figure 2

The frequency of each word of this verse in the entire Quran is always a multiple of 19. The word ***ism*** (name) is repeated 19 times; ***Allah*** (God), 2698 (19 X 142) times; ***Al-Rahman*** (The Most Gracious), 57 (19 X 3) times and ***Rahim*** (***The Most Merciful***), 114 (19 X 6) times.

Moreover, there is the same numeral relationship (number 19) between the letters of the first verse of the Quran and these same letters at the beginning of certain chapters (the Opening).

Below is a table which appears in a copy of a lecture delivered in Kuwait by Dr. Mohammed Rashad Khalif, entitled "***Number 19 - New Interpretation of the Quranic Miracle***."

Translation into English for the first three entries are as follows:

(1) The letter **nun** ن the opening of Chapter 64 is used 133 (7x19) times.

(2) The letter **kaf** ق is used as an opening for Chapters 50 and 57. The frequency of this letter in both chapters is 57 which equals a total of 114 (6x19).

(3) The letter **sad** ص is used in the openings of three chapters: 7, 19 and 38. The frequency of this letter totals 152 (8x19).

Number 19 - New Interpretation of the Quranic Miracle

Openings	Its Number	Sum of Its Letters	Multiples of 19's
ن	1	133	7 x 19
ق	2	114	6 x 19
ص	3	152	8 x 19
يس	1	285	15 x 19
طه	1	342	18 x 19
طس	6	494	26 x 19
طسم	6	1444	76 x 19
حم	7	2166	114 x 19
عسق	1	209	11 x 19
حم عسق	1	570	30 x 19
الم	8	26676	1404 x 19
الر	5	9707	511 x 19 الرعد
المص	1	5358	282 x 19
المر	1	1501	79 x 19
كهيعص	1	798	42 x 19
The Sum		49381	2599 x 19

Table 1

PART II
MATHEMATICS

The Arabs Search for Knowledge

Islam through the ***Quran, Sunnah*** and ***Seerah*** exhort the Muslim to search for knowledge.

On several occasions Prophet Mohammed admonished all Muslims to search for knowledge as we see from the following quotes:

> ***"The search for knowledge is an obligation laid on every Muslim,"***
>
> ***"If anyone pursues a path in search of knowledge, Allah will thereby make easy for him a path to paradise."***
>
> ***"Two greedy persons never attain satisfaction: he who is greedy for knowledge can never get enough of it, and he who is greedy for worldly goods can never get enough of them."***
>
> ***"The worst evil consists in learned men who are evil, and the best good consists in learned men who are good."*** **[4].**

In addition to the fact that the Prophet Mohammed, himself having implored his followers to seek knowledge from the cradle to the grave, Muslims were encouraged to study the sky and the earth to find proofs to their faith.

There was also a practical religious need for mathematics and astronomy. Such as developing trigonometric methods to find the direction to Mecca, towards which they turn daily in their prayers; using arithmetic and algebra to calculate inheritances and to count days and years. From Astronomy, Muslims could determine the beginning of Ramadan, the month of fasting, and other great holy days. Investigations in mathematics was given impetus by the teaching of Prophet Mohammed and the Quran. For example, a Latin translation of a Muslim arithmetic text was discovered in 1857 at the Library of the University of Cambridge. Entitled ***Algoritmi de numero Indorum***, the work opens with the words: "***Spoken has Algoritmi. Let us give deserved praise to God, our leader and defender.***"**[48]**

As a result of the Quran, Sunnah and Seerah, Muslims directed their attention to intellectual activities during the early days of Islam, approximately 700 CE, turning first to the practical sciences, such as mathematics and astronomy. Additionally, the Muslim mind has always been attracted to the mathematical sciences in accordance with the "abstract" character of the **doctrine of Oneness** which lies at the heart of Islam. (Surah 6:19; Surah 112:1-4) **[45].** The mathematical sciences have traditionally included astronomy, mathematics itself and much of what is called physics today.

Antiquity and the European Renaissance

The Translations

Translations of antiquity began under Caliph Al-Mansur and were further developed under his grandson, Al-Ma'mun. Beginning in 762 CE, al-Mansur established his capital in Baghdad and the caliph Harun al-Rashid, established a library. Following Al-Rashid's construction of a library to house both original and Arabic translations, Caliph al-Ma'mun (who reigned from 813 to 833) founded a translations and research institute known as "The House of Wisdom."

Many different translators rendered scientific works from the Greek, Sanskrit and Persian into Arabic. Euclid's work on geometry entitled ***Book of Basic Principles and Pillars*** was the first Greek work to be translated into Arabic. Al-Hajjaj ibn Yusuf translations included the first six books of Euclid and the ***Almagesti*** written by Ptolemy. Among the many very important translations done by Hunayn b. Ishaq was Menelaos' ***Spherica***. His son, Quota bin Luqa translated Diophantos' ***Arithmetica*** and Al-Hajjaj bin Matar translated Euclid's ***Elements.***

As was pointed out earlier in this book under the discussion of *The Spread of Islam*, most of the important philosophical and scientific works of Aristotle, Plato, the Pythagorean school, major works of Greek astronomy, mathematics, medicine, the Elements of Euclid and the works of Hippocrates and Galen were rendered into Arabic by the Islamic community. In addition to translations of the Greek works, important works of astronomy, mathematics and medicine were translated from Patilavi (Persia) and Sanskrit (India).

As a result, Arabic became the most important scientific language of the world for many centuries and the depository of much of the wisdom and sciences of antiquity.

The Moslems translated these works because the structure of Islam itself is based upon the primacy of knowledge.

From 1050 onward Western science and mathematics benefited from Arabic materials and their translators.

In the course of the Muslim conquests the Hindu-Arabic numerals and their arithmetic were carried by the Moors through North Africa and into Spain. Arabic numerals were known to Gerbert [Pope Sylvester II] in the 10th century and he introduced these numerals in the place of the awkward Roman numerals.

John N. Crossley and Alan S. Henry in their article, ***"Thus Spake al-Khawarizmi: A Translation of the Text of Cambridge University Library Ms. Ii.vi.5"*** published in <u>Historia Mathematica, Vol. 17, No. 2, May 1990</u>, says:

> "*The process involved in using Hindu-Arabic numerals, as opposed to Roman numerals, acquired the name "algorism" in the West. It is generally agreed that the word "algorithm" comes from the name of the scholar Abu Jafar Muhammad ibn Musa al-Khwarizmi, who lived about 800-847 and used the Arabic language. The Oxford English Dictionary states that the word "algorithm" was originally spelled "algorism" in its English version about the 12th century or perhaps slightly earlier. It was only very much later that the world "algorithm," spelled with "th," became current. In the 12th century and for a long time thereafter the spelling "algorism," with an "s," meant the rules and procedures for using the nine Hindu-Arabic numerals 1,2,3,4,5,7,8,9 and the cypher (Arabic "sifr") 0, though the actual shapes of these characters were different in those days*."

Gilbert was followed by others such as Constantinus Africanus in the eleventh century. Africanus traveled for thirty years in Muslim lands and studied under Arab (Muslim) teachers and translated several Arabic works into Latin.

In the 12^{th} century the algebra of Al-Khwarizmi was translated into Latin by Gerard of Cremona. Among the more than seventy

Arabic books Gerard translated into Latin was Archimedes' ***Measurement of the Circle***. Arabic translations and knowledge passed from Spain into Lorrains, Germany, Central Europe and England.

Adelard of Bath was among the first of many scholars of England who traveled extensively in search of Arabic books. He translated works on Mathematics and Astronomy to include a Latin translation of Euclid's Elements from Muslim sources, Al-Khwarizmi's tables and other works on the abacus and astrolabe. His Quaestiones Naturales consists of 76 scientific discussions derived from Muslim sciences.

Chart 1 is a summary of important mathematical works translated from the Greek into Arabic.

Arabic Translations of Greek Mathematical Works

AUTHOR	TITLE	TRANSLATOR
Euclid	The Elements	Al-Hajjad b. Matar Ishaq b. Hunayn Thabit b. Qurra
Ptolemy	Almagesti	Al-Hajjaj ibn Yusuf
Archimedes	Sphere and Cylinder Measurement of the Circle Heptagon in the Circle The Lemmas	Ishaq b. Hunayn Thabit b. Qurra Thabit b. Qurra Thabit b. Qurra
Apollonios	The Conics	Hilal al-Himsi Ahmad b. Musa Thabit b. Qurra
Diophantos	Arithmetic	Qusta b. Luqa
Menelaos	Spherica	Hunayn b.Ishaq

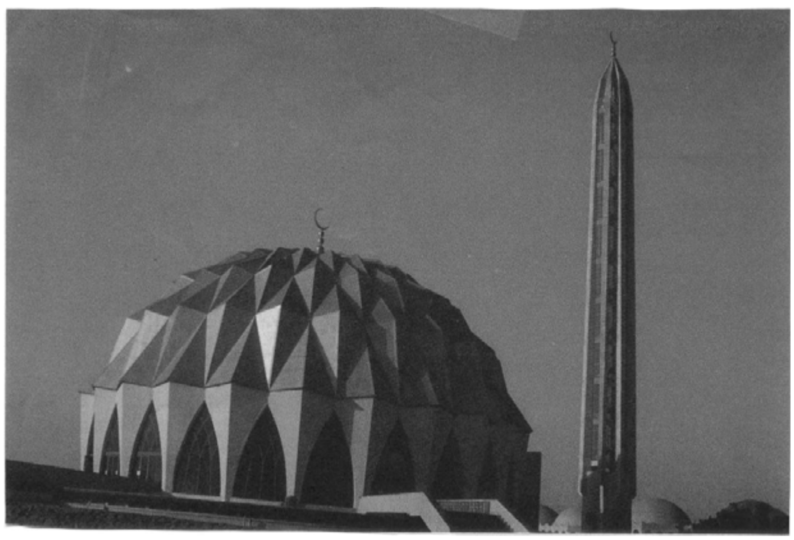
Chart 1

Geometry

Islamic Calligraphy and Geometric Constructions

Dominate in Islam are the elaborate geometrical designs executed in wood, tile or Mosaics. (See page 34). While there had always been a strong tradition of geometric design in the Middle East since the time of Egypt, the Islamic interest reflected its beliefs in their architecture - the gentle curves of the white arches of the Mosque School, where learning and a closer relationship to the Creator were inseparable pursuits.

On the next page you see a Mosque that reflects these gentle curves of the white arches in its architecture.



Gentle curves of white arches

Abu Nasr al-Farabi (870 CE), who taught philosophy in Baghdad and Syria, wrote a book on geometry with the title, ***A***

Book of Spiritual Crafts and Natural Secrets in the Details of Geometrical Figures.

Abul al-W'afa, incorporated all of al-Farabi's work in a book of his own, entitled, **On Those Parts of Geometry Needed by Craftsmen**.

Abul al-W'afa is best known for the first use of the ***tangent*** function and compiling tables of sines and tangents at 15' intervals. This work was done as part of an investigation into the orbit of the Moon, written down in Theories of the Moon. He also introduced the secant and cosecant and studied the interrelations between the six trigonometric lines associated with an arc. He and the prince Abu Nasr Mansur stated and proved theorems of plane and spherical geometry that could be applied by astronomers and geographers, including the laws of sines and tangents.

It was Abu Nasr Mansur's pupil, al-Biruni (973-1050), who produced a vast amount of high-quality work, and was one of the masters in applying these theorems to astronomy and to such problems in mathematical geography as the determination of latitudes and longitudes, the distances between cities, and the direction from one city to another, specifically, from a given city to Mecca.

It is in Abul al-W'afa's work entitled, **On Those Parts of Geometry Needed by Craftsmen**, in which problems on geometrical constructions that are possible using only a straightedge and a compass with one fixed opening are discussed and solved.

Problems using only a straightedge and compass will generally involve circles and polygons, which will invariably produce constructions involving isosceles triangles.

This suggests an answer to the question, why Al-Khwarizmi gave a proof of "*the sum of the squares of the two legs of an isosceles right triangle is equal to the square of the hypotenuse*" **[3]** rather than right triangles in general, as Pythagoras did. All Al-Khwarizmi needed do to prove the theorem of Pythagoras, was let the square HGRT inscribed in square ABCD (see Figure 3) have vertices ***not*** equal distant from the vertices of the larger square ABDC, so that:

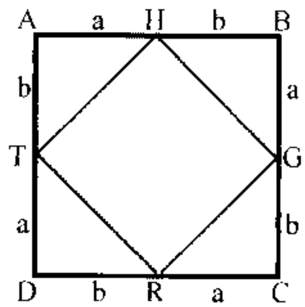


Figure 3

a is not necessarily equal to ***b***, and then instead of adding the areas of the triangles, (i.e., TA^2 = area of 2 triangles and AH^2 = area of 2 triangles and TH^2 = area of four triangles) subtract the area of the triangles from the area of ABDC which would leave the area of THGR:

$$(a+b)^2 - 4(\frac{1}{2}ab) = c^2$$
$$a^2 + b^2 = c^2.$$

Al-Khwarizmi's algebra contained other geometrical ideas. In his section on ***Mensuration*** he calculated areas of triangles, parallelograms, pyramids, and circles. He says:

> ***"If you multiply the diameter of any circle by itself, and subtract from the product one-seventh and half one-seventh of the same, then the remainder is equal to the area of the circle."***

That is, $\pi = 3\frac{1}{7}$.

So we see how Islam's first pillar of faith, Shahada, with the accompanying calligraphy involving geometric constructions is connected to the extensive development in geometry.

Shahada was the impetus for the Arab Muslim's work in geometry, beginning with Euclid's work on geometry entitled ***Book of Basic Principles and Pillars***, **[48],** the first Greek work to be translated for students in Arab lands. Shahada was the motivation for Thabit ibn Qurrah, his grandson Ibrahim ibn Sinan (909-946), Abu Sahl al-Kuhi (c 995), and Alhazen resulting in the solution of problems involving the pure geometry of conic sections and including the areas and volumes of plane and solid figures from which they were formed.

Translations of various works began under Al-Mansur and were further developed under his grandson, Al-Ma'mum. A prince with a fine intellect, a scholar, philosopher, and theologian, Al-Ma'mum was instrumental in the discovery and translation of the works of ancient people. During the reign of Harun Al-Rashid, Al-Hajjaj ibn Yusuf translated into Arabic several Greek works. Among these translation were the first six books of Euclid and the ***Almagest***. [The name 'Almagest' is a Latinized version of the Arabic title ***Almagesti***.] The Almagest, written by Claudius Ptolemy of Alexandria, was the most outstanding ancient Greek work on astronomy. **[18].** The work of the Muslims in the application of geometry to the solution of algebraic equations suggests they were the first to establish the close interrelation of algebra and geometry. This was a leading contribution toward the later development of analytic geometry. It was during the ninth and tenth centuries that the Arab Muslims gave to Europe the first information about Euclid's ***Elements.*** **[37].**

Thabit ibn Qurra (836-911 CE) of Harran, Mesopotamia, is often regarded as the greatest Arab geometer. **[19].** He carried on the work of Al-Khwarizmi and translated into Arabic seven of the eight books of the conic section of Apollonious. **[6].** He also translated certain works of Euclid, Archimedies, and Ptolemy which become standard texts. **[13].**

The Arab Contribution to Mathematics

Archimedes' original work on the regular heptagon had been lost, but the Arab translation by Thabit ibn Qurra proves the Greek manuscript still existed at the time of translation. Carl Schoy found the Arabian manuscript in Cairo, and revealed it to the Western public. It was translated into German in 1929. **[35].** As the signs of mathematical awakening of Europe appeared in the 1200's, the Greek classics were available for translation. As the Christian monks made contact with Muslim universities in Spain, opening the way to the Renaissance, Euclid's ***Elements*** were translated again, but this time from Arabic to Latin.

The Arab Contribution to Mathematics

Trigonometry

The era from 700 CE to 1100 CE was a period during which the Arab Muslims came into possession of a spirit of discovery and scholarship which distinguished the era as the "Muslim Arab Renaissance" and led to setting the stage for the European Renaissance.

The study of science and mathematics was kept alive by the Muslims during a period when the Christian world was in the "Dark Ages." The Muslims had more than a passing interest in the works of earlier civilizations. One can see this interest in the fact that they translated virtually all known information of their day into Arabic. Al-Hajjaj ibn Yusuf translated Ptolemy's work from Greek into Arabic and based their trigonometry on his theorem of half-chords.. However, the Arabs made two important and superior differences: (1) They employed the ***sine*** where Ptolemy used the chord and (2) they wrote their trigonometry in algebraic instead of geometric form.

The perpendiculars drawn from the end point of the radii, to the original directions, form segments which correspond to the half-chords of Ptolemy. Half-chord in Arabic is ***jiba*** and became confused with ***jaib***. ***Jaib***, had nothing to do with the length of a half-chord, but rather meant, "*The opening of a garment at the neck and bosom*." European mathematicians translated the Arabic word "***jaib***" by the Latin word "***sinus***", meaning "***bosom***" or "***fold***." And "***sine***" was derived from "***sinus***."

Rene Taton writes, "*The Muslims were vastly superior to the Greeks and Indians in the area of trigonometry*." **[44].** Here, Islam exerted its influence by connecting the commandments of the Quran with the developments in trigonometry in two ways: (1) They needed the use of trigonometry to present a clear model of the heavens and (2) its relationships to the Muslim's mode of life, which included the problem of finding the direction of Mecca and prayer times. In studying the mystery of God and the relationship between heaven and earth, the Muslims directed their attention to spherical trigonometry, and Al-Battani became their chief proponent.

Al-Battani

Mohammed ibn Jabir Abu Abdullah Al-Battani was born in Battan, Mesopotamia in 850 CE and died in Damascus in 929 CE.**[1]** He was an Arabian prince, governor of Syria, and is held as one of the greatest astronomers of Islam. He is responsible for a number of important discoveries in astronomy.

His well-known discovery is the remarkably accurate determination of the solar year as being 365 days, 5 hours, 46 minutes and 24 seconds, which is very close to the latest estimates. His excellent observations of lunar and solar eclipses were used by

Dunthorne in 1749 to determine the acceleration of motion of the moon.

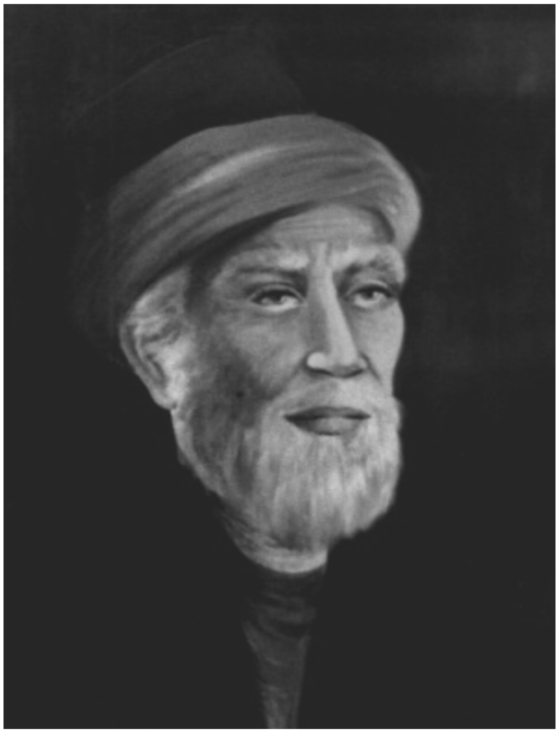
He also provided very neat solutions by means of orthographic projection for some problems of spherical trigonometry. Al-Battani determined with remarkable accuracy the obliquity of the ecliptic, the length of the seasons and the true and mean orbit of the sun.

In mathematics, he was the first to replace the use of Greek chords by sines, with a clear understanding of their superiority. He also developed the concept of cotangent and furnished their table in degrees.

He wrote a number of books on astronomy and trigonometry. His most famous book was his astronomical treatise with tables, which was translated into Latin in the 12th century and flourished as De scienta stellerum — De numeris stellerum et motibus.

His treatise on astronomy was extremely influential in Europe until the Renaissance, with translations available in several languages. His original discoveries both in astronomy and trigonometry were of great consequence in the development of these sciences. Copernicus in his book De Revolutionibus Orbium Clestium expresses his indebtedness to Al-Battani.

Al-Biruni was another among those who laid the foundation for modern trigonometry. **[39].**



Al-Biruni

As a philosopher, geographer, and astronomer, Al-Biruni was not only a mathematician but a physicist as well. Taki Ed Din al-Hilali considers Al-Biruni to be "*one of the very greatest scientists of all time*." Six hundred years before Galileo, Al-Biruni had discussed the possibility of the earth's rotation around its own axis**. [33].**

Al-Biruni was born outside of the city of Khwarizmi, present day Khiva, a city of Uzbekistan in 972 C.E. and died in 1048 C.E. in Ghazna (Afghanistan) after a forty-year illustrious career. Many scholars recognize in al-Biruni the exemplification of the true Islamic spirit in scientific inquiry. Trained initially as a mathematician, al-Biruni ventured into the fields of chemistry, astronomy, history, geography, and pharmacology. He proved to be the authority, par excellence, of classical Islamic knowledge. His mastery of Arabic, Persian, Sanskrit, and Greek gave him access to the treasures of ancient civilizations and a means to grasp their ``truths".

He said: "My experience in the study of astronomy and geometry and experiments in physics revealed to me that there must be a Planning Mind of Unlimited Power. My discoveries in Astronomy showed that there are fantastic intricacies in the universe which prove that there is a creative system and a meticulous control that cannot be explained through sheer physical and material causes."

Al-Biruni carried out geodesic measurements **[15]** and determined the magnitude of the earth’s circumference. **[38].**

The Arab Contribution to Mathematics

From all his contributions to mathematics and physics, Al-Biruni is best known in the World of Islam for fixing the direction to Mecca in mosques all over the world. [5]. And, if it had not been for the influence of Islam in his life, he probably would never have acquired the knowledge of mathematics and science sufficient to accomplish this feat.

Mecca is the birthplace of the Prophet Mohammad and the most sacred spot in the Islamic world. Every year, during Ramadan, more than one million Muslims make a pilgrimage, or hajj, to Mecca.

The city is surrounded by mountains and valleys which during Hajj season turn to towns of white tents for pilgrims.

Non-Muslims are strictly banned to enter Mecca or the nearby pilgrimage towns such as Mina, Arafat, and Mozdalifah.

The Ka'aba, a windowless cube-shaped building, fifty-feet high, in the courtyard of the mosque, is believed by Muslims to have been built by the Hebrew patriarch Ibrahim (Abraham). It is draped in black and gold material which is traditionally renewed annually.

In the southeastern corner of the Ka'aba is the black cornerstone, cast down by God to Adam after he was removed from the Garden of Eden. This is a symbol of God's reconciliation with mankind.

According to Islamic tradition, Muslims around the world must face Mecca during their daily prayers. This direction toward Mecca is called al-qibla.

If we know where we are (point Z); where Mecca is (point M) and where the North Pole (point P) is, then we have a spherical triangle (ZPM), as shown in Figure 4:

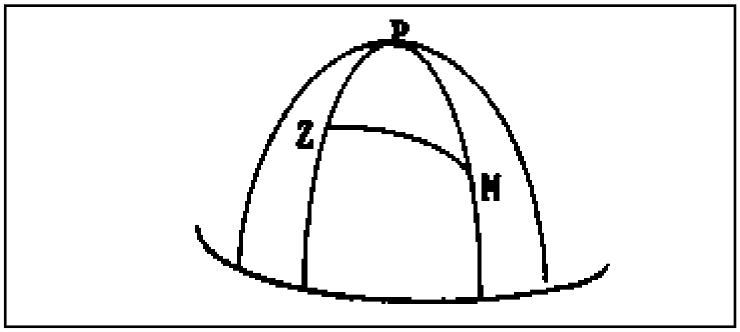


Figure 4

Angle PZM is the local azimuth of Mecca, the ***qibla***. To find the ***qibla*** we must know two sides and the included angle in triangle ZPM.

Al-Biruni stated, justified and then applied the Sine Theorem (law of sines) to a series of spherical triangles to determine the ***qibla***. The law of sines (Sine Theorem) states: For a plane triangle, the sides of a triangle are proportional to the sines of the opposite angles. If the angles are A, B, C, and the lengths of the sides opposite these angles are a, b, c, this law is:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Expressed in modern notation Al-Biruni stated, justified and applied the law of sines to a series of spherical triangles as appears in Table 2 below:

Table of Spherical Trigonometric Laws of Sines and Cosines

Plane Trigonometry	Spherical Trigonometry
Law of Sines	
$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$	$\frac{\text{Sin}\,a}{\sin A}=\frac{\text{Sin}\,b}{\sin B}=\frac{\text{Sin}\,c}{\sin C}$
Law of Cosines	
$a^2 = b^2 + c^2 - 2bc\text{Cos}A$	$\text{Cos}\,a = \text{Cos}b\text{Cos}c + \text{Sin}b\text{Sin}c\text{Cos}A$
$b^2 = c^2 + a^2 - 2ca\text{Cos}B$	$\text{Cos}\,b = \text{Cos}c\text{Cos}a + \text{Sin}c\text{Sin}a\text{Cos}B$
$c^2 = a^2 + b^2 - 2ab\text{Cos}C$	$\text{Cos}\,c = \text{Cos}a\text{Cos}b + \text{Sin}a\text{Sin}b\text{Cos}C$

where for the plane triangle a, b, and c have units of length, and for the spherical triangle a, b and c are the angles subtended at the center of the sphere by the great circle arcs, e.g.

$$a = \frac{\text{length of arc a}}{\text{radius of sphere}}$$

Note that the spherical triangle formulae reduce to the plane triangle formulae when a, b and c (in radians) are all much less than 1.

Table 2

The Arab Contribution to Mathematics

Time of Prayers

In addition to finding the "direction of prayer," spherical trigonometry was used for the purpose of determining the time of the daily Muslim prayers.

These times were defined in terms of the position of the sun relative to the horizon, hence the composition of tables for prayer times was an exercise in spherical applied astronomy that was used for civil and astronomical purposes as well.

This science of time-keeping (***'ilm al-miqat***) gave rise to a group of astronomers who were associated with major mosques and whose duty it was to tell the ***muezzin*** when to call the faithful to prayer.

In the case of the afternoon prayer, for example, the convention used is when the shadow of an upright rod in the ground equals the length of its noon shadow plus the length of the rod. For each degree of longitude of the sun, from the altitude of the sun at the beginning of prayer until sunset determines the permitted time for this prayer, as illustrated in figure 5:

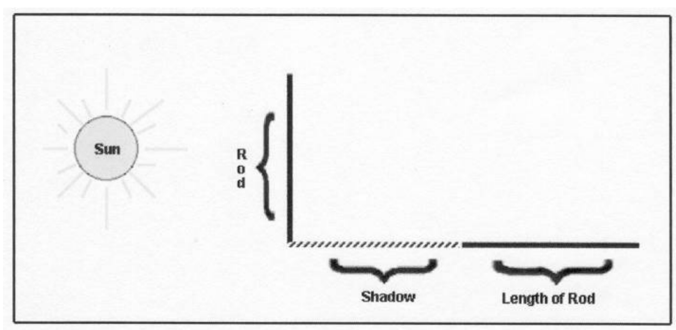


Figure 5

The Arab Contribution to Mathematics

Number Theory

The Zakat (The Poor Due or The Tithe)

One of the principal topics to be studied by the Muslims has always been arithmetic - which has been proliferated over the centuries into what we today refer to as number theory.

The calculation of the Zakat, Islam's influence on the use of arithmetic is commanded by Islam:

> **"And they are ordered naught else than to serve Allah, in sincere devotion to Him, being monotheists, and to perform prayer perfectly and to pay the Zakat. That is true religion." [surt-al-Bayyinah]**

As explained in Part I (Duties) of this book, the religious duties (**'ibadat**) of the Muslims center on the five pillars of Islam. The third pillar of Islam is "Paying The Poor Due."

Allah ordained every Muslim who possesses a certain amount of property to pay annually the Zakat (poor due) of his possessions to the poor, or to the other categories mentioned in the Quran.

The minimum amount of gold liable to payment of Zakat is 20 ***miskals*** of gold (a miskal is a weight equals 4.68 grams) and the minimum amount of silver is 200 dirhams (a dirham is a weight that equals 3.12 grams) or an equivalent sum of current money to this amount.

There is also a minimum amount for goods of commerce liable to payment of Zakat. As for cereals and grains, its minimum amount is 300 ***saas*** (a ***saa'*** is a cubic measure used by Arabs). The minimum amount for real estates prepared for sale should be estimated in accordance with its value, but if the real estate is prepared for lease, the estimation is considered in accordance with its rent.

The annual amount of Zakat fixed on gold, silver and goods of commerce is 2.5%.

Diophantus of Alexandria, compiled the first text on number theory. He collected existing problems and invented new ones, and entitled his treatise, ***Arithmetica***. Number Theorists today, still refer to this specialty as "***Arithmetic***." For 400 years Alexandria had been the intellectual capital of the civilized world. The library where Diophantus had spent his life-time compiling the ***Arithmetica***, was the largest in the world and contained among its vast holdings six hundred years of mathematical progress. Only six of the 13 books comprising the ***Arithmetica*** survived it's destruction in 389 CE.

For the next 400 years, a few adroit individuals in India copied the formulae from the surviving Greek manuscripts and added new elements to mathematics. The Hindus began to use Sanskrit characters as their numbers, which was the practice the Greek used. It was from these characters our present day Hindu-Arabic numerals evolved.

The Hindu-Arabic Numeration System

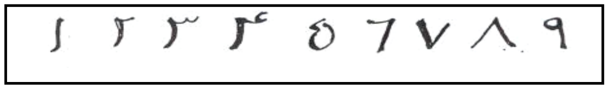
The most often asked question by the general public regarding the history of Arabian mathematics concerns the origin of our numeral system.

It is impossible to give a definitive answer to this question because it is not possible to trace precisely the development of the Hindu-Arabic numeral system.

The Hindu numerals were known to the early Arabic mathematicians and references to the Hindu numerals are mentioned by Arab Muslims as early as the 9th century.

The Arab Contribution to Mathematics

In his book on the ***Principles of Hindu Reckoning***, **Folio 268a**, Kushybar ibn Labban (ca 971-1029 CE), gives the nine numerals (left to right) as:



(Sanskrit related)
Figure 6

The above numerals are related to the Sanskrit characters the Hindus began to use around 400 CE, following the practice of the Greeks who used letters of their alphabet as their numerals.

It was the Muslim's that gave us zero. The Hindu's used it as a place holder but the Muslim's treated it as a number. Also, the earliest Muslim zero is contained in a manuscript that predates the earliest Hindu manuscript which contains a "zero." Our word "zero" derives from the Arabic ***sifr,*** which was Latinized into "zephirum." The word ***sifr*** itself was an Arabic translation of the Sanskrit word *sunya*, meaning "empty." An alternate Medieval translation of ***sifr*** into "cifra" led to our modern English "cipher."

There are many words in the mathematical vocabulary which are borrowed from Arabic words, such as the following:

The Arab Contribution to Mathematics

Mathematical vocabulary borrowed from Arabic

Latinized Name	Arabic Name
Algebra	al-Jabr
Algorism, Algorithm	Al-Khawarizmi
Atlas	Atlas
Azimuth	Al-sumut
Cipher, Zero	Sifr
Cosine	Cosine
Nadir	Nadir, Nazir
Sine	Sine
Tangent	Tangent
Zenith	Cenit

Chart 2

The Muslim's used a dot '•' for zero. It was also the Muslim’s that gave us the modern notation for common fractions and the use of decimal fractions. We call our numerals Arabic because the principles in the two systems are the same and the variations that took place over time resulted in today’s numeration system.

From the beginning of the 10th century, the Arabs were using the following numerals which are still used in Saudi Arabia today:

٩	٨	٧	٦	٥	٤	٣	٢	١	•
9	8	7	6	5	4	3	2	1	0

(Early Arabic Numerals)
Figure 7

The above symbols were finally transformed into today’s modern Hindu-Arabic numerals (reading right to left) **zero-0, one-1, two-2, three-3, four-4, five-5, six-6, seven-7, eight-8, nine-9.**

The Arab Contribution to Mathematics

The details of how the symbols were transformed into today's modern Hindu-Arabic numerals are missing and unimportant.

The important thing was that in the 10th century, Gerbert of Aurillac, from France, learned the numerals from the Moors of Spain and introduced the system to the West through his teaching positions in churches and schools in Europe.

The Arabian Muslims had replaced the primitive Greek symbols and cumbersome Roman numerals with the numeral system Europe adopted by the 13th century and is used universally today.

Among the Muslim mathematicians, Abu-Yusef Ya'qub ibn Ishaq Al-Kindi contributed the most to Arithmetic **[7].**

Following is a list of the eleven texts he wrote on the subject:

1. An Introduction to Arithmetic.
2. Manuscript on the Use of Indian Numbers.
3. Manuscript on Explanation of the Numbers mentioned by Plato in his politics.
4. Manuscript on the Harmony of Numbers.
5. Manuscript of Unity from the Point of View of Numbers.
6. Manuscript on Elucidating the Implied Numbers.
7. Manuscript on Prediction from the Point of View of Numbers.
8. Manuscript on Lines and Multiplication with Numbers .
9. Manuscript on Relative Quantity.
10. Manuscript on Measuring of Proportions and Times.
11. Manuscript on Numerical Procedures and Cancellation.

Al-Kindi

Yaqub ibn Ishaq Al-Kindi (Alkindus) was born of noble Arabic descent at Kufa, approximately 800 A.D., and flourished in Iraq under the caliphs al-Ma`mun (813-833) and al-Mu'tasim (833-842). He died in 873 A.D. Al-Kindi was not only a mathematician, but the first outstanding Islamic philosopher, known as "the philosopher of the Arabs." He was also a physicist, astronomer, physician, geographer and an expert in music.

He was a prolific writer, having written more than 270 works. A large number of his books were translated into Latin by Gherard of Cremona. In addition to the 11 books he wrote on Arithmetic, he wrote extensively on Astronomy, Geometry, Medicine, Physics, Philosophy, Logic, Psychology, and Music.

Al-Kindi also wrote various monographs concerning tides, astronomical instruments, rocks, and precious stones.

Most of his books are no longer in existence, but those that have survived, are reportedly significant in both content and scholarship. His work on shadows and chords of circles led to further development of mathematics and his work on springs and the specific weight of precious stones and metals led to further development of physics. His accurate calculations of latitude and longitude were based on the actuality that the earth rotated on its axis. He invented a plane projection of the sphere that could be used to produce a map of a hemisphere. In 1030 he wrote ***al Qanun al-Mas'udi*** which contains a collection of 23 observations of equinoxes beginning with observations by Hipparchus and Ptolemy and ending with two observations which he made himself. Cardano considered him as one of the twelve greatest minds during the Middle Ages.

Al-Karkhi of Baghdad **(**Abu Bekr Mohammed ibn Al-hosain Al-karkhi) was the most scholarly and the most original writer of arithmetic. He was born in Karkh, a suburb of Baghdad, and died in the decade 1019-29. A Muslim mathematician at Baghdad, he wrote on arithmetic, algebra, and geometry. Two of his works are known: ***Al-Kafi fi al-Hisab***, which means "*Essentials of Arithmetic*" and ***Al-Fakhri***, which was the name of his friend, the grand vizier in Baghdad at that time. **[36].**

W.W. Rouse Ball, reports that:

> ***"Al-Karkhi gave expressions for the sums of the first, second, and third powers of the first n natural numbers; solved various equations, including some of the forms*** $ax^{2p} \pm bx^{p} \pm c = 0$ ***; and discussed surds, showing for example, that*** $\sqrt{8}+\sqrt{18}=\sqrt{50}$ **."** **[8].**

Diophantus' ***Arithmetica*** discussed the solution of linear and quadratic equations, but he lacked an easy system of numerals to build his solutions. The Muslims brought a new number language from the East and the classical mathematics from Greece to Fibonacci, who studied in Muslim schools and in 1202 CE introduced Arabian numerals to Europe. **[25].** By 1453 CE, when the Turks ransacked Constantinople, they discovered the manuscripts which survived the destruction of the Alexandria library had been carefully preserved by the Muslim scholars. Diophantos' ***Arithmetica*** had been translated into Arabic by Qusta ibn Luqa of present-day Lebanon, probably in the ninth century.**[9].**

Diophantus was now in the hands of Byzantine Scholars headed for the desk of Pierre de Fermat who was already in possession of the Arabic numeral system. As Fermat read Diophantus, he would write comments in the margins of the book. While studying Book II of the ***Arithmetica***, Fermat read the observations, problems and solution concerned with Pythagoras' Theorem and Pythagorean triples. Euclid's Elements had been translated again, but this time by al-Hajjaj bin Matar, from Arabic to Latin, which Fermat read, and thus was familiar with Euclid's proof that there are an infinite number of Pythagorean triplets. As he played with Pythagoras' equation, he wrote an equation, which was very similar to Pythagoras' equation, but had no solution at all. Instead of considering the equation: $\mathbf{x^2+y^2=z^2}$, Fermat wrote: $\mathbf{x^3+y^3=z^3}$, which turns Pythagoras' equation with an infinite number of solutions into an equation with no solutions. So in the margin of his **Arithmetica**, next to Problem 8, Fermat wrote a note that said there are no whole numbers which satisfy:

$$\mathbf{x^n+y^n=z^n, \quad where \;\; n \geq 3.}$$

Then Fermat added the comment:

> "*On the contrary, it is impossible to separate a cube into two cubes, a fourth power into two fourth powers, or, generally, any power above the second into two powers of the same degree: I have discovered a truly marvelous demonstration which this margin is too narrow to contain*." (Fermat, ***Oeuvres***, III, p. 241)

Over 300 years later, this was the equation Andrew Wiles read about at ten years of age and gave the world a proof for 30 years later.

The West, and indeed the whole world is familiar with this story of "Fermat's Last Theorem." However, the world is ***not*** familiar with the following episode in the history of number theory which involved Fermat. Before Fermat reached Problem 8 of Book II, where he wrote his now so very famous quotation, he made a discovery concerning amicable numbers. Amicable numbers are pairs of numbers such that each number is the sum of the divisors of the other number. The Pythagoreans knew that 220 and 284 are amicable or "friendly" numbers, and are given credit for discovering this fact. But this was known long before the Pythagoreans. Before mathematicians discovered **"Amicable"** numbers, the Bible used them four times. The first time is in Geneses 32:14.

Notice that Amicable, is an adjective. Defined by Webster as: *Characterized by or showing goodwill; peaceable.*

And that amicable numbers are defined by Webster as: *Two numbers, each of which is equal to the sum of all the exact divisors of the other except the number itself.*

For example, the numbers 220 and 284 are amicable numbers, because:

If m = {1, 2, 4, 5, 10, 11, 20, 22, 44, 55, 110} then m|220
and
1 + 2 + 4 + 5 + 10 + 11 + 20 + 22 + 44 + 55 + 110 = 284

If n = {1,2,4,71,142} then n| 284
and
1 + 2 + 4 + 71 + 142 = 220

[Note: The symbol "|" means "divides"; for example 55|220 (read "fifty-five divides 220") means that 55 is an exact divisor of 220.]

We arrive at the number 220 by **"amalgamating"** 200 females and 20 males.

The second, third and fourth time these numbers are used we find in:

Ezra 8:20:

> **Also of the Nethinims, whom David and the princes had appointed for the service of the Levites, two hundred and twenty Nethinims: all of them were expressed by name.**

I Chronicles 15:6

> **Of the sons of Merari; Asaiah the chief, and his brethren two hundred and twenty:**

Nehemiah 11:18

> **All the Levites in the holy city were two hundred fourscore and four.**

These three places are **"amicably"** related: All are connected to the tribe of Levi, whose name derives from the wish of Levi's mother to be **"amicably"** related to his father:

(Gen. 29:34):

> **And she conceived again, and bare a son; and said, Now this time will my husband be joined unto me, because I have born him three sons: therefore was his name called Levi.**

The above three references have to do with (1) Ezra 8: - the returning exiles (Levites) to Jerusalem, the rebuilding of the walls of Jerusalem with (Nehemiah 11:18) the 284 Levites living there with (I Chronicles 15:6) 220 descendants of Merari. As follows:

- Ezra 8: The list of Ezra's company of returning exiles, and their arrival at Jerusalem.
  Vs. 20: David had appointed 220 Nethinims for the service of the Levites.

- Nehemiah: The rebuilding of the walls of Jerusalem.

  Ch. 11: The call for people to dwell in Jerusalem.

  Vs. 18: 284 of these people were Levites.

- I Chronicles 15: The Ark brought to Jerusalem.

Vs. 2: None to carry the Ark except the Levites.
Vs. 6: The sons of Merari, 220 Merarites, descendants of Merari living in Jerusalem after the rebuilding.

In summary, the connection between the numbers 220 and 284 and all the above scriptures is the fact they are "amicably" related. All are connected to the Tribe of Levi, whose name derives from the wish of Levi's mother to be "amicably" related to his father Jacob. The West still thinks that "*no other amicable numbers were identified until 1636 when Fermat discovered the pair 17,296 and 18,416*." **[41].** In his number-1 bestseller, Singh goes on to say:

> ***"Descartes discovered a third pair (9,363,584 and 9,437,056) and Leonhard Euler went on to list sixty-two amicable pairs."* [41].**

During the last half of the 9th century, over 700 years before Fermat was born, the Arabian Muslim, Thabit ibn Qurra (826 CE - 901 CE), who is particularly noted for his translations of works from Greek to Arabic by Euclid, Archimedes, Appollonius, Ptolemy and Entocious **[36],** had already identified the amicable number pairs 17,296 and 18,416. As a matter of fact, he did a great deal more! A remarkable formula for amicable numbers is credited to him **[11].** The formula is as follows:

If p, q and r are prime numbers, and if they are of the form,
$p = 3\bullet 2^n - 1$, $q = 3\bullet 2^{n-1} - 1$, $r = 9\bullet 2^{2n-1} - 1$,
Then p, q and r are distinct primes and $2^n pq$ and $2^n r$ are a pair of amicable numbers.

For Example: For n = 2, p =3(4)-1=11, q=3(2)-1=5, r=9(8)1=71 and 2^2(11)(5) = 220 and 2^2(71)=284.

Notice that when n=4 you get: p =47, q=23, r=1151 and 2^4(47)(23) = 17 296 and 2^4(1151)=18 416.

Without the many translations of Thabit ibn Qurra, the number of Greek mathematical works known today would be smaller. For example, we would have only the first four of the seven books or more of Apollonius' ***Conics***. In adherence to Prophet Mohammmed's admonition:

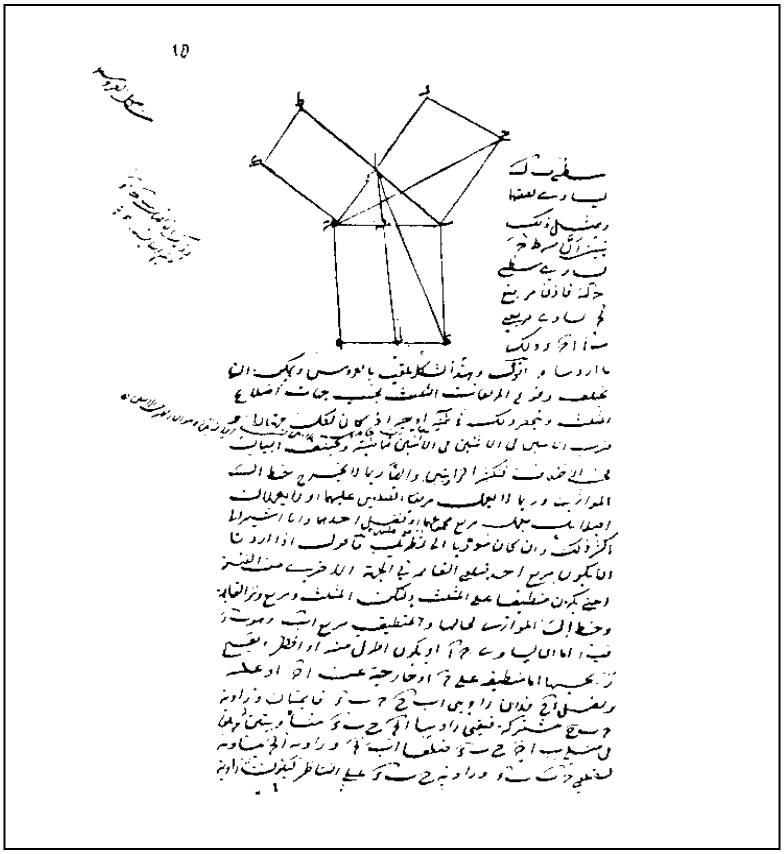
> ***"He who is asked about something he knows and conceals it will have a bridle of fire put on him on the day of resurrection." -Prophet Mohammed*** **[4].**

Thabit ibn Qurra did not conceal his knowledge of the Greek mathematical classics, but widely disseminated, through his translations, modifications and generalizations of mathematics findings which created an interest in particular areas of mathematics, such as amicable numbers, that these topics formed a continuing tradition in Islam. Kamal ad-Din al-Farisi (who, by the way, translated Brahmagupta's astronomical work into Arabic) gave the pair 17,926 and 18,416 as an example of Thabit's rule, and in the 17th century Muhammad Baqir Yazdi gave the pair 9,363,584 and 9,437,056.

The Arab Contribution to Mathematics

Thabit ibn Qurra

Thabit ibn Qurra was born in Harran (present day Turkey) in northern Mesopotamia in 836 CE, and died in Baghdad in 901 CE. He was an Arab mathematician, physician, and philosopher, and a representative of the flourishing Arab-Islamic culture of the 9th century. He went to Baghdad and obtained a thorough mathematical and philosophical training. Through the influence of several men who were trusted advisors of the 'Abbasid caliph al-Mu'tadid, Thabit secured appointment as a court astronomer in Baghdad, where he spent the remainder of his life writing mathematical, philosophical, and medical works. He applied arithmetical terminology to geometrical quantities, and studied several aspects of conic sections, angle trisection and magic squares. A number of his computations aimed at determining the surfaces and volumes of different types of solids.



Pythagorean Theorem in Qurra's Translation of Euclid

Courtesy of The National Council of Teachers of Mathematics. Classics in Mathematics Education, A Series, "The Pythagorean Proposition."

The Arab Contribution to Mathematics

It is not the intention of the author to try to do justice to the Muslim contributions to mathematics in this book, but rather, as stated in the preface, to show the influence Islam has had on the Muslim contribution to mathematics. However, in the limited reading and research that I did in producing this work, it is my sense that there exist manuscripts, some of which have yet to be translated into English, that would yield an abundance of work done by the Arabic Muslims in the field of number theory. For example, we know that the Muslim mathematician, Al-Karkhi gave expressions for the sum of the first, second, and third powers of the first ***n*** natural numbers as follows: **[5].**

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}.$$

We also know that the introduction of the line separation between the numerator and denominator of a common fraction is due to the Muslims. To denote a fraction in the Muslim method, one writes three-fourths as $\frac{3}{4}$, the same way it is still written. Additionally, it is to Muslim mathematicians that credit is due for the first use of the decimal fractions. The Arabic word for fraction, ***al-kasr***, is derived from the stem of the verb meaning “to break.” **[29].**

Astronomy & The Calendar

Ramadan and Hajj

From Astronomy, Muslims could determine the beginning of Ramadan, the month of fasting, and other great holy days.

Hajj **-** Pilgrimage is the fifth and last pillar of Islam. Once in a lifetime every Moslem of either sex, who is financially and physically able, is supposed to undertake during Ramadan a holy visit to Mecca.

Umrah **-** is the lesser pilgrimage to Mecca and may be made individually and at any time.

To Mecca: Moslems Only

The year 1 of the Islamic era begins with July 16, 622 of the Gregorian calendar. There is a perfect correlation between the Julian and the Lunar calendar:

Hijra, The year of the Prophet Mohammed, or 1A.H. (Anno Hegirae), the Lunar Year, began July 16, 622 A.D., The Gregorian solar year.

One lunar month is equal to the cycle between two new moons encompassing:

29 days + 12 hours + 44 minutes + 2.8 seconds

Thus, a Lunar year = 354 days and $\frac{11}{30}$ of a day. In 30 years this equals 11 days (30 X $\frac{11}{30}$ days = 11 days)

Letting x = Gregorian year and y = Hijra year we can write a set of equations:

$$\begin{cases} x = 622 + \frac{32}{33} y \\ y = \frac{33}{32}(x - 622) \end{cases}$$

The above equations give us the corresponding years between the two. For example, the Hijra year corresponding to the year 2000 A.D. is:

$$y = \frac{33}{32}(2000 - 622) = \frac{33}{32}(1378) = 1421 \text{A.H.}$$

The mathematics that the Muslims inherited from the Greeks made many problems important to Islam scholars extremely complicated or impossible. For example, the ***division of an estate***. This topic, known in Arabic as ***'ilm al-fara'id*** **[10, 21]** (the science of the legal shares of the natural heirs). In Islam, if a man dies his wife receives 1/8 of the estate (1/4 if there are no children) and each male child receives twice what the female child receives. If there are only female children, 2/3 of the estate is divided among the girls. If there is only 1 female child, 1/2 the estate goes to the female child. All of this is further complicated by the inheritance of 1/6 of the estate, each, for the parents of the deceased and the inheritance of the brothers of the deceased if there were no male children. **[17].**

Abu Abdullah Mohammad Ibn Musa al-Khawarizmi's algebra served as a model for later writers in its application of arithmetic and algebra to the distribution of inheritances according to the complex requirements of Muslim religious law.

It was the search for more accurate, comprehensive, and flexible methods that led Mohammed ibn Musa al-Khwarizmi (c. 825 CE) to make modifications in the algebra inherited from the Greeks and on to epoch-making original contributions in algebra.

In his ***Arithmetica***, **[16]** Diophantus of Alexandria (c. 275 CE) gave a method of reducing indeterminate equations to simple equations that can be solved directly or take the form of determinate equations.

A single equation is indeterminate if it has more than one variable and has an infinite number of solutions. An example would be: $6x + 8y = 46$. When the coefficients are integers and it is required to find solutions in a restricted class of numbers, for example, positive integers, then the equation is called Diopantine equations. Diophantus was the first to pose and solve problems that called for solutions in integers or rational numbers.

In solving these determinate equations Diophantus followed definite methods. For example, in solving $ax^2 + bx = c$, he multiplied both sides by "a" in order to make the first term a perfect square **(a^2x^2).**

The Arab scholars translated and used what the Greeks achieved in algebra to make original contributions themselves which proved to be epoch-making achievements. Al-Khowarizmi made modifications in Diophantus' methods. For example, in solving the same equation, $ax^2 + bx = c$, he divides both sides of the equation by **"a",** in order to reduce the first term to a perfect square **(x^2),** which is the method used today.

While engaged in astronomical work at Baghdad and Constantinople, Al-Khowarizmi wrote the algebra which brought him fame, ***Al-jabr ma-al-muqabala*** (*The Science of Cancellation and Reduction*) written in 820 CE. **[3].** ***Al-jabr ma-al-muqabala*** is devoted to finding solutions to problems posed by the requirements of Islam and practical problems which the Muslims encountered in daily life concerning matters of inheritance, legacies, partition, lawsuits and commerce.

In his book Al-Khwarizmi first demonstrates his methods and then turns his attention to solving those problems posed by Islamic requirements. Al-Khwarizmi, in his introduction to ***Al-jabr ma-al-muqabala*** explains in his introduction why he came to write his algebra text:

> "That fondness for science, by which God has distinguished the Imam al-Ma'mun, the Commander of the Faithful ... has encouraged me to compose a short work on calculating by *al-jahr* and *al-muqabala* , confining it to what is easiest and most useful in arithmetic, such as men constantly require in cases of inheritance, legacies, partition, ..." **[3].**

A Latin translation of this text became known in Europe under the title **Al-Jabr**. Thus, the Arabic word for reduction, **al-Jabr**, became the word ***algebra***. **[22].**

He cites a problem on the topic ***'ilm al-fara'id***, (inheritance) as followings:

> ***A woman dies, leaving her husband, a son, and three daughters and bequeathing to a stranger one-eighth and one-seventh of her capital;"*** **[3, pages 89&90].**

The ***Shariah*** (Islamic Law) declares the husband receives 1/4 of the estate and the son twice that of a daughter.

Al-Khwarizmi's solution is as follows:

> **Each daughter receives 1/5 of 3/4 = 3/20, of the residue, and the son, 6/20. Since the stranger takes 1/8 + 1/7 = 15/56 of the capital, the residue = 41/56 of the capital, and each 1/20th share of the residue = 1/20 x 41/56 = 41/1120 of the capital. The stranger, therefore, receives 15/56 = (15x20)/(56x20) = 300/1120 of the capital. [3, page 90].**

In **The Supplement of Arithmetic** by the Iraqi traveler, Abu Mansur al-Baghdadi, there is a problem to pay the Zakat on 7586 dirhams. **[8].** The dirham was divided into sixty ***fulus***, the plural of ***fils***.

Al-Baghdadi calculates the total due on 7586 dirhams, as follows:

"7586

From the first place we remove 1, which we make 40, and then remove 6 from the 40. This is the zakat due on 6 dirhams and it is 6 parts of (the 40 into which we have divided) a dirham.
Note: 2.5% of 40 fulus = 1, so, 6 fulus is the zakat due on 6 dirhams.

Thus, of the 40 (fulus) there remains 34 (fulus). This we put under the fils that has remained in the units place."

7585

34

We must now calculate 1/40 of the 80 that arises from the 10's place, to obtain 2, which we subtract from the five in the units place.

7583

34

In the 100's place there is 500, on which the zakat due is 12 1/2." (500 divided by 40 equals 12 1/2.).

Of the 40 parts into which we have divided the dirham, 1/2 is 20, so when we subtract this from 34, there remains 14 parts. Also, 12 from 83 leaves 71, so there now remains:

7571

14

So, for the 1st year we have left 7396 dirhams and 14 fulus.
Subtract from 7585 40
7396 14
189 d 26 fulus = 7586 fulus."

Since 7586 dirhams = 303440 fulus, and $\mathbf{2\frac{1}{2}\%}$ of 303440 = 7586 fulus, we see the calculation is correct.

Since one dirham = 60 fulus, not 40, the base-40 fraction, convenient to use, must now be converted into sexagesimal fractions:
7586/40 = 7586 X (3/2)/(40X(3/2))=7586/40 = 7586(3/2)/40(3/2) = 11379/60 = 189.65 which is $\mathbf{2\frac{1}{2}\%}$ of 7586. **[9, pp 65-67].**

In developing his algebra, Al-Khwarizmi transformed the number from its earlier arithmetical character as a finite magnitude into an element of relation and of infinite possibilities. It can be said that the step connecting arithmetic to algebra is in essence a step from "being" to "becoming" or from the static universe of the Greeks to the dynamic ever-living, God-permeated one of the Muslims. Solomon Gandz wrote the following concerning Al-Khwarizmi:

> ***"Al-Khwarizmi's algebra is regarded as the foundation and cornerstone of the sciences. In a sense, Al-Khwarizmi is more entitled to be called 'the father of algebra' than Diophantus because Al-Khwarizmi is the first to teach algebra in an elementary form and for its own sake, Diophantus is primarily concerned with the theory of numbers."*** **[20].**

<u>**The Arab Contribution to Mathematics**</u>

al-Khwarizmi

Muhammad ibn Musa al-Khwarizmi, was born about 770 C.E. and died in Baghdad in 850 C.E. He was a Muslim mathematician and astronomer whose major works introduced Hindu-Arabic numerals and the concepts of algebra into European mathematics.

In addition to being a mathematician and astronomer, Al-Khawarizmi was a geographer. In his book on geography, **The Image of the Earth,** he developed a map of the Islamic world much superior to that known from the work of Ptolemy by correcting Ptolemy's exaggerated length of the Mediterranean Sea.

His work on algebra <u>**Kitab al-jabr wa al-muqabalah**</u> ("The Book of Restoring and Balancing"), was translated into Latin in the 12th century and originated the term algebra (al-jahr). He established the subject in a systematic form and developed it to the extent of giving analytical solutions of linear and quadratic equations.

Another work on Hindu-Arabic numerals is preserved only in a Latin translation, **Algoritmi de numero Indorum** ("Al-Khwarizmi Concerning the Hindu Art of Reckoning"). In this work he explained the use of zero, a numeral of fundamental importance developed by the Arabs and, additionally, developed the decimal system. It was from the title of this book (*Algoritmi* for Al-Khwarizmi) that the term *algorithm* originated.

In addition to introducing the Indian system of numerals (now generally known as Arabic numerals), he developed at length several arithmetical procedures, including operations on fractions. It was through his work that the system of numerals was first introduced to Arabs and later to Europeans, through its translations in European languages.

He developed in detail trigonometric tables containing the sine functions. The development of astronomical tables by him was a significant contribution to the science of astronomy, on which he also wrote a book. He is also reported to have collaborated in the degree measurements ordered by Mamun al-Rashid which were aimed at measuring the volume and circumference of the earth.

The influence of Khawarizmi on the growth of science, in general, and mathematics, astronomy and geography in particular, is well established in history. He has been held in high repute throughout the centuries since then.

Muhammad ibn Musa al-Khwarizmi influenced mathematical thought to a greater extent than any other mediaeval writer. He was one of the great mathematicians of all times. **[24].**

In his book, Al-Khwarizmi gives a proof of: The sum of the squares of the two legs of an isosceles right triangle is equal to the square of the hypotenuse. He writes: **[3]**

Observe, that in every rectangular triangle the two short sides, each multiplied by itself and the products added together, equal the product of the long side multiplied by itself. The proof of this is the following. We draw a quadrangle, with equal sides and angles ABCD. We divide the line AC into two moieties in the point H, from which we draw a parallel to the point R. Then we divide, also, the line AB into two moieties at the point T, and draw a parallel to the point G. then the quadrate ABCD is divided into four quadrangles of equal sides and angles, and of equal area; namely, the squares AK, CK, BK, and DK. Now, we draw from the point H to the point T a line which divides the quadrangle AK into two equal parts: thus there arise two triangles from the quadrangle, namely, the triangles ATH and HKT. We know that AT is the moiety of AB, and that AH is equal to it, being the moiety of AC; and the line TH joins them opposite the right angle. In the same manner we draw lines from T to R, and from R to G, and from G to H. Thus from all the squares eight equal triangles arise, four of which must, consequently, be equal to the moiety of the great quadrate AD. We know that the line AT multiplied by itself is like the area of two triangles, and AK gives the area of two triangles equal to them; the sum of them is therefore four triangles. But the line HT, multiplied by itself, gives likewise the area of four such triangles. We perceive, therefore, that the sum of AT multiplied by itself, added to AH multiplied by itself, is equal to TH multiplied by itself. This is the observation which we were desirous to elucidate. Here is the figure to it:

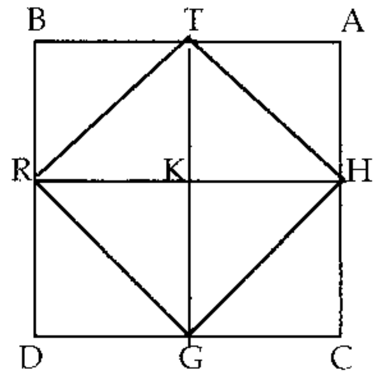


Figure 8

Since Mohammed ibn Musa Al-Khwarizmi was the founder of the Muslim school of mathematics, the subsequent Muslim and early medieval works on algebra were largely founded on his algebraic treatise. Al-Khwarizmi's work plays an important role in the history of mathematics, for it is one of the main sources through which Arabic numerals and Muslim algebra came to Europe.

This tradition of service to the Islamic faith was the enduring feature of mathematical work in Islam and one which, in the eyes of the Muslim's, such as al-Mahani, justified the study of secular learning. Al-Mahani (c. 860), the next Muslim contributor to algebra, took up Archimedes' problem of cutting the sphere into two segments, the ratio of which is equal to a given ratio, and rendered it such celebrity that the equation $x^3 + a^2b = cx^2$ came to be known as the **al-Mahani's equation**. Even though he found no solution for it, his work led Abu Ja'far al-Khazin (c. 960) of Khorasan to a solution for it by means of the intersection of conic sections.

Arab mathematicians accomplished some significant results in seeking a solution for the fourth degree equation. Among them were Abul W'afa, whose method was lost and Mohammad ibn al-Leit (c. 1000), who developed a solution by the intersection of conic sections. **[30].** Al-Karkhi of Baghdad (1020 CE), the most scholarly and the most original writer of arithmetic, who was mentioned earlier, also made a marked contribution to algebra. His work in algebra includes operations of algebraic quantities, roots, equations of the first and second degree, indeterminate analysis, and solution of problems. **[30].** Al-Karkhi contributed rational solutions to certain special equations of degree higher than two and a method for approximating the solution to linear equations. The Muslims not only created algebra, which was to become the indispensable instrument of scientific analysis, but they laid the foundations for methods in modern experimental research by the use of mathematical models. These are but a few of the more outstanding developments in algebra that resulted directly from the efforts of Muslim mathematicians influenced by Islam. The Greeks expressed their algebra in geometrical terms and as a result their achievements were confined to a few analytical solutions.

The Arabians influenced by their Semite origin, were endowed with minds that valued practicality. Additionally, they were influenced by Islam which involved problems of religious duties, such as the calendar, hours of daily prayers and determination of exact directions toward Mecca.

Chart 3, below, is a summary of only those works referred to in this book.

WORKS OF ARABIC MUSLIM MATHEMATICIANS (Referenced in this Book)

NAME	CENTURY	WORKS
Al-Farabi, Abu Nasr	9th	A Book of Spiritual Crafts & Natural Secrets in the Details of Geometrical Figures
Al-Kindi, Abu Yusef Ya'qub ibn Ishaq	9th	Eleven texts on Arithmetic and Number Theory
Qurra, Thabit ibn	9th	Number Theory
Al-Khwarizmi, Mohammed ibn Musa	9th	Algebra Al-jabr ma-al-muqabala
Al-Mahani	9th	Algebra
Al-Battani, Mohammed ibn Jabir	9th /10th	Spherical Trigonometry
Al-Khazin, Abu Ja'fer	10th	Solved Archimedes Problem: Cutting the sphere into two segments.
L-Wafa, Abul	10th	On Obtaining Cube and Fourth Roots and Roots Composed of These Two.
Labban, Kushybaribn	10th	Principles of Hindu Reckoning
Al-Biruni, Abu l-Rayhan	10th /11th	Spherical Trigonometry Physics Fixing the direction to Mecca.
Al-Karkhi, of Baghdad	11th	1) Al-Kafi fi al-Hisab 2) Al-Fakhri
Al-Baghdadi, Abu Mansur	11th	The Supplement of Arithmetic
Al-Leit, Mohammad ibn	11th	Quartic Equation

Chart 3

Conclusion

The people of Europe and the United States are unaware of Islam's great relevance to their own history. Most mathematicians are unfamiliar with Islam's relevance to the history of mathematics, and even unfamiliar with most of the mathematical contributions made by the Arabs of the geological Arabian Peninsula.

This is due in part to the fact that Western historians have purposely overlooked and minimized the importance of the Islamic society and culture, particularly in Spain beginning with the 8th century.

All of this lack of information and misinformation continues to permeate Western Civilization because of ethno-centric, religious prejudice and an absence of easily obtainable sources. The preservation of intellectual contact of ancient Greek, Roman, Persia and Indian civilizations has been recognized, but the Arabs original contributions have not.

Pope Urban (Claremont, France), Nov. 26, 1095) exerted the Christians to:

> ***"enter upon the road to the Holy Sepulcher, wrest it from the wicked race, and subject it."***

One hundred fifty thousand men (Crusaders) responded by meeting at Constantinople and attacking the Arabs mercilessly, destroying and devastating all that lay in their path. Libraries, with ancient and original works were burned and hundreds of thousands of the inhabitants massacred.

The fate of the Islamic Arab Empire was sealed by the invasion and destruction of the Mongols led by Hulagu Khan. By the time the Ottoman Turks had risen to power (13th century) the Islamic Empire had been so weakened that the Turks had no difficulty in expanding their power to include the entire Arab World. The Turks, ruling in the name of Islam, turned Muslim against Christian, Sunni against Shi'a and Kurd against Armenian.

With the destruction of their great system of irrigations, that made the region fertile; orchards razed to the ground; centers of learning destroyed and people massacred by the hundreds of thousands followed by the occupation of the Turks who considered the Arab Muslims inferior, the Islamic Arab Empire was excluded from all activities that contribute to a genuine civilization. The Arab Muslims were made to perform menial tasks during the European Renaissance, the Eighteenth Century Enlightenment and during the Industrial Revolution.

The Ottoman Empire lasted until the end of WWI with the defeat of Germany and its allies which included Turkey.

With the ending of WWI in 1918, Great Britain became the dominant force in the region.

In 1913 Abdulaziz Al-Saud fought the Turks at the Hassa Province (Hofuf) on the Arabian Gulf. He needed the ports in the region to obtain armaments and food. In April, 1913, Abdulaziz eliminated the Turks from the Hassa region and all territory from Kuwait to Qatar was now under his control. Saudi control was extended to the outskirts of Hail in 1917. At the end of the war Great Britain was determined to limit Arabs self-rule. Because the United States declared its support for Arab self-determination, good relations were established between the Arabs and the U.S.

King Abdulaziz Al-Saud was successful in unifying the Kingdom of Saudi Arabia by 1932 and he is known today in Saudi Arabia as the "father of the country."

The discovery and production of oil in 1939 added to its strategic importance and economic progress. Because Saudi Arabia is the birthplace of Prophet Mohammed and the location of the Holy Muslim Shrines in Madinah and Mecca, it is the spiritual center for all Muslims.

Contrary to the Western view, Saudi Arabia is in an energy/economic, military strategic and spiritual position to become the Islamic force and influence, to lead the Islamic Arabian geological peninsula to enlightened influences and reform movements that recognize the necessity for adapting the old teaching to the present era.

The Arab Contribution to Mathematics

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GLOSSARY

Akkad – The first recorded empire in history in the area known as Babylonia.

Allah - The one and only true God; the creator of all things.

Al-qibla - In spherical trigonometry, if we know where we are (point Z); where Mecca is (point M) and where the North Pole (point P) is, then we have a spherical triangle (ZPM), The direction toward M, Mecca, is called ***al-qibla***. See Figure 4.

Amir - Leader or commander

Arab – Any Arabic-speaking person.

Arabia – Today Arabia means the country of Saudi Arabia, but historically it was the entire eastern part of the geological peninsula which includes Bahrain, Iraq, Kuwait, Oman, Qutar, Saudi Arabia, United Arab Emirates, and Yemen.

Arabian - Refers to an inhabitant of the geological peninsula [See map].

Ausa'a – A verb meaning "*to make wider, more spacious, to extend, to expand*."

C.E. **-** Common Era [A.D.]

Caliph - successor (Khalifa or "Caliph") to Mohammed, assuming leadership of the community of believers.

Hadith - Sayings of the Prophet Mohammed

Hajj - Pilgrimage; the fifth and last pillar of Islam.

'ilm al-fara'id -The division of an estate (the science of the legal shares of the natural heirs).

‘ilm al-miqat - The science of time-keeping.

Imam – Leader of the congregational prayer, Salat, that the Muslims offer five time a day.

Ism - Arabic word for “name”

Jaib/jiba - Half-chord in Arabic is ***jiba*** and became confused with ***jaib***. ***Jaib***, had nothing to do with the length of a half-chord, but rather meant, “*The opening of a garment at the neck and bosom*.”

Jihad - Holy War(s)

Ka’aba - The first house of worship built for mankind. It was originally built by Adam and later on reconstructed by Abraham and Ismail. It is a cubed shaped structure based in the city of Mecca, to which all Muslims turn to in their daily prayers.

Madinah - The first city-state that came under the banner of Islam. It is where the Prophet Mohammed’s masjid and grave are situated.

Masjid - A place of worship and salat. The life of the early Muslims used to revolve around the masjid. It is called in “mosque” in English.

Mecca - Mecca is the site of the *Ka’aba*, the most sacred spot in the Islamic world, and it is the direction to which Muslims must turn to say their daily prayers.

Poor Due - *Zakat* - Almsgiving or Purifying Tax - To pay annually 2.5% tithe of one's net savings on which a year has passed as a religious duty and purifying sum to be spent on poorer sections of the community. The Third Pillar of Faith.

Qibla - The same as al-qibla.

Quran - Muslims believe the Quran is the infallible, inerrant, Word of God. They believe it is authentic, original and complete. The Quran is a record of the exact words revealed by God through the Angel Gabriel to the Prophet Mohammed.

Ramadan - The holy month of prescribed fasting for the Muslims. It was during this month that the Quranic revelations began.

Rasul - Messenger of God

Ratim - The Most Merciful

Salat - The five obligatory prayers that a Muslim must perform every day.

Saum - Fasting

Seerah - Biography of Prophet Mohammed

Shahadah - Declaration of faith: "I testify that there is no god but Allah and I testify that Mohammed is the Messenger of Allah."

Shariah – Islamic Law

Shirk – the worst and only unpardonable sin is *shirk*, joining or associating of other gods with the true God. (Allah).

Sunnah- The examples of the Prophet's life, what he said, did, implemented, how he implemented.

Taharah - A state of legal purity

Umrah – The lesser pilgrimage to Mecca and may be made individually and at any time.

Zakat - The obligatory tax that every Muslim must give. It is one of the five pillars of Islam. The zakat is used to provide for the poor and destitute.

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